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Predicted Effects of By-Pass Flows on Regenerator Performance

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Summary.—Regenerative heat, or heat and mass, exchangers, termed regenerators, are used to provide waste heat recovery in air conditioning, gas turbine and various industrial plants. By-pass flows, such as leakage, affect the heat transfer effectivity of these regenerators, and cause contaminant transfer.

The effects of by-pass flows on the overall heat and mass transfer effectivities of a balanced regenerator are examined. A simple model of a regenerator with by-pass flows is formulated, from which the necessary equations of mass and energy conservation are derived, enabling general expressions for the overall effectivities to be found.

The types of by-pass flows occurring in rotary and switched-bed regenerators are explained. Non-dimensional curves are drawn showing the effects of some particular cases of by-pass flows on overall effectivity. The significance of the results is discussed, with particular reference to air conditioning applications.

LIST OF SYMBOLS

C	cross by-pass mass flow rate as proportion of flow through one side of regenerator; C_i into inlet of side considered, C_o into outlet, C_{ox} from outlet
c_a	specific heat at constant pressure of dry air
h	enthalpy of moist air per unit mass of dry air
h_g	enthalpy of water vapour per unit mass of dry air at temperature t, assumed to be independent of any temperature variation
m	mass flow rate
m_r	balanced mass flow rate through a side of the regenerator
n	cycle frequency for switched-bed regenerator
p	pressure in fluid flow
S	side by-pass mass flow rate as proportion of flow through one side of regenerator
t	temperature
V_1, V_2, V_3, V_4	trapped volumes shown in Fig. 2
w	water content (humidity ratio) of air/water vapour mixture, mass of water vapour per unit mass of dry air
η_o	overall regenerator effectivity, including the effects of by-pass flows
η_r	regenerator effectivity
ρ	fluid density
L, R	denote the left side and right side of the regenerator respectively (Fig. 3)

Subscripts

$1, 2, 3, 4, a, b, c, d, e, f, i, o$	denote locations on the regenerator model (Fig. 3), except where applied to V and η
h, w, t	used with η_o and η_r to denote that the effectivity is for the transfer of enthalpy, moisture or temperature respectively

1.—INTRODUCTION

Regenerative heat, or heat and mass, exchangers, termed regenerators, are used to provide waste heat recovery in air conditioning, gas turbine and various industrial plants. The regenerator transfers heat and fluid components between two fluid streams by passing the streams alternately through a porous matrix. Portions of the fluid streams usually by-pass the matrix. These by-pass flows change the overall effectivity of the regenerator for the transfer of heat and fluid components. This effect makes it important to minimise these flows in gas turbine regenerators (Ref. 1). The effect is also important in air conditioning applications, both where contaminant transfer needs to be controlled, and in regenerative evaporative cooling cycles since the performance of these is affected by moisture transfer in the regenerative heat exchanger.

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Regenerators take two forms, rotary and switched-bed, which are illustrated by those used in air conditioning. In the rotary regenerator (Fig. 1 and Ref. 2) the matrix is rotated through the two fluid streams, while in the switched-bed regenerator (Fig. 2 and Ref. 3) the matrix is divided into two equal parts and the fluid streams passed alternately through each part by means of a valve. By-pass flows in a rotary regenerator are due to pressure differences causing leakage through seals, and much design ingenuity is required to minimise these flows (Ref. 1). Leakage through the valve provides one by-pass flow in the switched-bed regenerator, and others are described in Section 2.

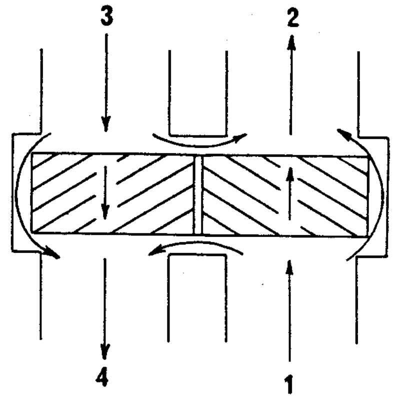

Fig. 1.—Rotary regenerator with by-pass flows.

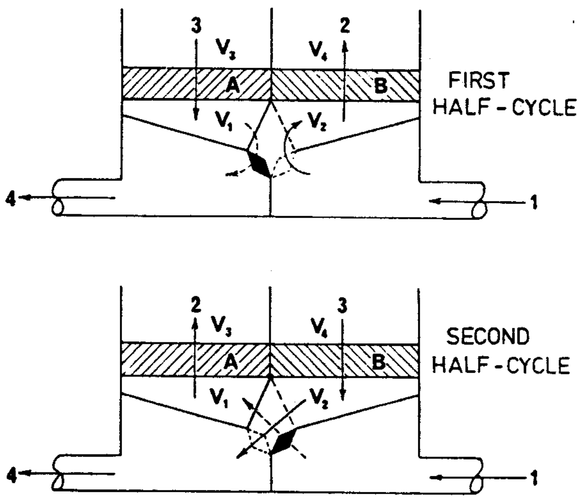

Fig. 2.—Switched-bed regenerator. Full and broken lines show sections through the different halves of the regenerator length.

A simple model of a regenerator with by-pass flows is formulated in Section 2, from which equations of mass and energy conservation are derived and solved in Section 3. The effects of various types and combinations of by-pass flows are then presented in Section 4.

A counterflow regenerator which is balanced, that is with equal mass flow rates in each fluid stream through the matrix, is considered. With balanced flows the regenerator is most effective, and hence in its most desirable mode of operation. In actual systems, where by-pass flows are a small proportion of through flows, the results obtained will be applicable also to the case of equal flow rates in the streams approaching the regenerator.

The effect of leakage flows occurring in a gas turbine regenerator on its overall effectivity has been derived in Ref. 4. This result is a particular case of the more general results presented here, since the same assumptions are made in the derivation.

2.—MODEL

The model considered is shown in Fig. 3, and consists of a counter-flow regenerator surrounded by by-pass flow paths. The two fluid streams through the regenerator have equal mass flow rates m_r and define the two sides of the regenerator denoted L and R. This definition follows directly from the arrangement of the rotary regenerator (Fig. 1), and is made clear for the switched-bed regenerator by Fig. 2. Two forms of by-pass flow are shown on Fig. 3.

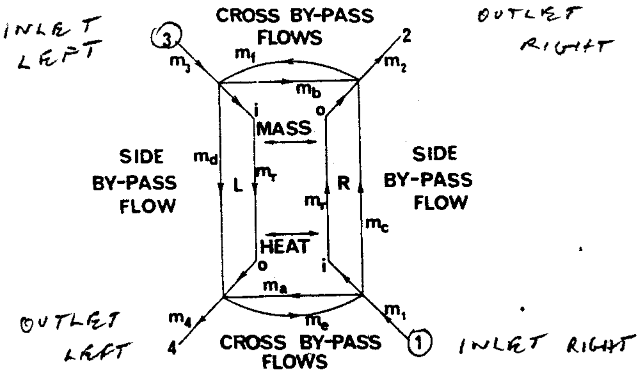

Fig. 3.—Transfer and mixing model of a regenerator with by-pass flows.

Side by-pass flow spans a regenerator side. It occurs only in a rotary regenerator and, being caused by pressure difference, necessarily has the same direction as the through fluid stream on the same regenerator side.

Cross by-pass flow may enter and leave both the inlet and outlet of a regenerator side. In a rotary regenerator, where by-pass flows are caused by pressure differences across seals, cross by-pass flow into the inlets of both sides simultaneously cannot occur. In a switched-bed regenerator (Fig. 2), leakage through the valve due to pressure difference causes cross by-pass flow into the outlet of one regenerator side (flow m_a on Fig. 3); external mixing of fluid streams 2 and 3 in Fig. 2 provides a cross by-pass flow into the inlet of the same side (flow m_f on Fig. 3); this is the important side for the application described in Ref. 3.

Cross by-pass flow also occurs in a switched-bed regenerator due to the fluid contained in the spaces between the matrix and the valve. This fluid is trapped when the valve is switched. For example, when switching from the first to the second half cycle valve positions shown in Fig. 2, fluid at state 4 is trapped below matrix portion A in volume V_1 and is mixed with fluid at state 1 (flow m_e in Fig. 3). Also fluid at state 1 is trapped below matrix portion B in volume V_2 and mixed with fluid at state 4 (flow m_a in Fig. 3). Similarly by-pass flows m_b and m_f are caused by the fluid volumes above the matrix V_3 and V_4. These volumes comprise the fluid in the ducts above the matrix and the fluid discharged from the ducts then drawn in when the flow direction changes. Hence the volumes V_3 and V_4 depend on duct design and ambient disturbances. The cross by-pass flow rates are given by

$$m_a = m_e = \rho\,(V_1 + V_2)\,n$$
$$m_b = m_f = \rho\,(V_3 + V_4)\,n$$

where the symbols have the meanings listed.

Cross by-pass flow from the inlet of a regenerator side and into the outlet of a side (m_a or m_b) and side by-pass flow (m_c or m_d) result in direct mixing of streams downstream of the regenerator. In contrast, cross by-pass flow from the outlet of a regenerator side and into the inlet of a side (m_e or m_f) results in direct mixing of streams upstream of the regenerator, thus affecting the through flows. The effects of these by-pass flows on the regenerator effectivities is evaluated below using conservation equations at each region where streams mix.

3.—ASSUMPTIONS AND ANALYSIS

3.1 Overall enthalpy effectivity:

Conservation of energy for the regenerator gives

$$m_r(h_{Lo} - h_{Li}) = m_r(h_{Ri} - h_{Ro}) \qquad \text{.................(1)}$$

so that the enthalpy effectivity of the regenerator is given by

$$\eta_{rh} = \frac{h_{Lo} - h_{Li}}{h_{Ri} - h_{Li}} = \frac{h_{Ro} - h_{Ri}}{h_{Li} - h_{Ri}} \qquad \text{.................(2)}$$

For the case of a balanced sensible heat regenerator η_{rh} is also the regenerator heat transfer effectivity as defined in Ref. 5.

The overall enthalpy effectivities for the left and right sides are respectively

$$\eta_{ohL} = \frac{h_4 - h_3}{h_1 - h_3}\,;\quad \eta_{ohR} = \frac{h_1 - h_2}{h_1 - h_3} \qquad \text{.................(3)}$$

The mass conservation equations at the four mixing regions shown in Fig. 3 may be written as

$$\begin{aligned} m_1 &= m_a + m_c + m_r - m_e \\ m_2 &= m_b + m_c + m_r - m_f \\ m_3 &= m_b + m_d + m_r - m_f \\ m_4 &= m_a + m_d + m_r - m_e \end{aligned} \qquad \text{.......................(4)}$$

A by-pass flow entering a through fluid stream is assumed to mix perfectly with the stream. This assumption is far from valid for cross by-pass flow into the inlet of a side of the regenerator. The flow through the regenerator would then actually be non-uniform in state, and the effect of such non-uniformity has not been analysed. Should its effect on regenerator effectivity be linear then the effective state of the flow through the regenerator may be obtained by averaging, a procedure equivalent to the assumption of perfect mixing of by-pass and through flows. Dunkle and Maclaine-cross (Ref. 2) suggest that this is a satisfactory procedure for the design of rotary regenerators. The enthalpy conservation equations for the mixing regions are then

$$\begin{aligned} m_r h_{Ri} &= m_1 h_1 - m_a h_{ai} - m_c h_{ci} + m_e h_{eo} \\ m_r h_{Ro} &= m_2 h_2 - m_b h_{bo} - m_c h_{co} + m_f h_{fi} \\ m_r h_{Li} &= m_3 h_3 - m_b h_{bi} - m_d h_{di} + m_f h_{fo} \\ m_r h_{Lo} &= m_4 h_4 - m_a h_{ao} - m_d h_{do} + m_e h_{ei} \end{aligned} \qquad \text{..............(5)}$$

where subscripts i and o represent regenerator inlet and outlet states respectively.

As a first approximation for this analysis it is assumed that the by-pass flows enter their respective passages with the same state as the adjacent duct flow, and that the by-pass flows leave their passages with the same state as at entry. These assumptions give

$$\begin{aligned} h_1 &= h_{ai} = h_{ao} = h_{ci} = h_{co} \\ h_3 &= h_{bi} = h_{bo} = h_{di} = h_{do} \\ h_{Ro} &= h_{fi} = h_{fo} \\ h_{Lo} &= h_{ei} = h_{eo} \end{aligned} \qquad \text{....................(6)}$$

If the by-pass flows are now expressed as proportions of the flow rate through either side of the regenerator, one obtains

$$\begin{aligned} m_a &= C_{oa} m_r\,, & m_d &= S_d m_r \\ m_b &= C_{ob} m_r\,, & m_e &= C_{ie} m_r \\ m_c &= S_c m_r\,, & m_f &= C_{if} m_r \end{aligned} \qquad \text{.......................(7)}$$

where the coefficients of m_r are the by-pass flow ratios; C signifies cross by-pass flow; S side by-pass flow; o by-pass flow into outlet of a side; i by-pass flow into inlet of a side; and a, b, c, d, e and f the location of the particular by-pass flow (Fig. 3).

Using (4), (5), (6) and (7) it follows that

$$(h_{Li} - h_3) = C_{if}(h_{Ro} - h_3) \qquad \text{.....................................(8)}$$
$$(h_{Ri} - h_1) = C_{ie}(h_{Lo} - h_1) \qquad \text{.....................................(9)}$$
$$(1 - C_{ie})(h_{Lo} - h_4) = S_d(h_4 - h_3) + C_{oa}(h_4 - h_1) \qquad \text{......(10)}$$
$$(1 - C_{if})(h_{Ro} - h_2) = S_c(h_2 - h_1) + C_{ob}(h_2 - h_3) \qquad \text{.....(11)}$$

We obtain from (2), (3), (8), (9), (10) and (11) general relationships for the overall enthalpy effectivity for the left and right sides of the regenerator, and these are given in (12) and (13).

$$\eta_{ohL} = \frac{\eta_{rh}(1 - C_{ie}) + C_{oa}\dfrac{(1 - C_{ie}\,\eta_{rh})}{(1 - C_{ie})} + \dfrac{C_{if}}{(1 - C_{if})}(1 + C_{oa} - C_{ie})(1 - \eta_{rh})}{(1 - C_{ie} + S_d + C_{oa})\left[1 + (1 - \eta_{rh})\left\{\dfrac{C_{ie}}{(1 - C_{ie})} + \dfrac{C_{if}}{(1 - C_{if})}\right\}\right]} \qquad \text{..........(12)}$$

$$\eta_{ohR} = \frac{\eta_{rh}(1 - C_{if}) + C_{ob}\dfrac{(1 - C_{if}\,\eta_{rh})}{(1 - C_{if})} + \dfrac{C_{ie}}{(1 - C_{ie})}(1 + C_{ob} - C_{if})(1 - \eta_{rh})}{(1 - C_{if} + S_c + C_{ob})\left[1 + (1 - \eta_{rh})\left\{\dfrac{C_{ie}}{(1 - C_{ie})} + \dfrac{C_{if}}{(1 - C_{if})}\right\}\right]} \qquad \text{..........(13)}$$

3.2 Overall moisture (or other fluid component) effectivity:

We can write water vapour conservation equations for the water content in the flows, similar to (5), and these are

$$\begin{aligned} m_r w_{Ri} &= m_1 w_1 - m_a w_{ai} - m_c w_{ci} + m_e w_{eo} \\ m_r w_{Ro} &= m_2 w_2 - m_b w_{bo} - m_c w_{co} + m_f w_{fi} \\ m_r w_{Li} &= m_3 w_3 - m_b w_{bi} - m_d w_{di} + m_f w_{fo} \\ m_r w_{Lo} &= m_4 w_4 - m_a w_{ao} - m_d w_{do} + m_e w_{ei} \end{aligned} \qquad \text{........(5a)}$$

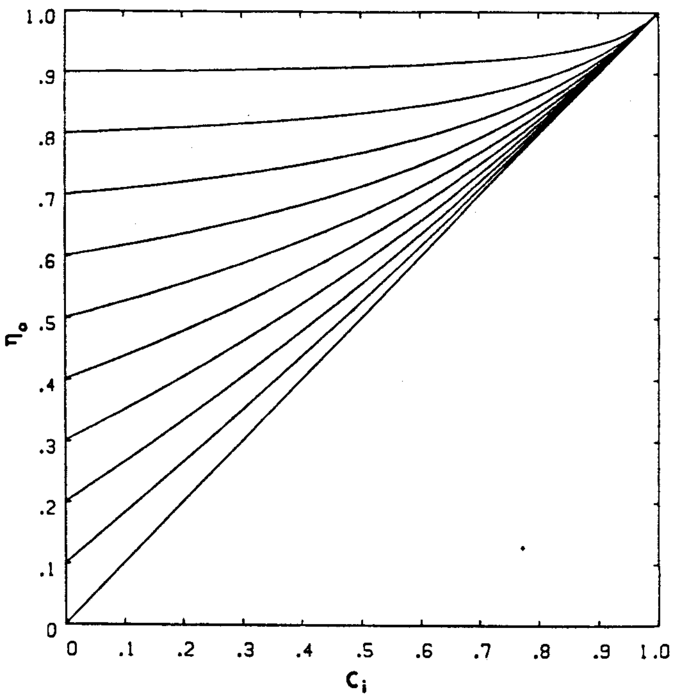

Fig. 4.—Effect of cross by-pass flow into inlet, C_i, on the overall effectivity of a regenerator side, η_o, for a range of values of regenerator effectivity, η_r, given by $\eta_o = \eta_r$ at $C_i = 0$.

The water vapour conservation equation within the counterflow exchanger, assuming that there is moisture transfer between the two flows, is analogous to (1), and is

$$m_r(w_{Lo} - w_{Li}) = m_r(w_{Ri} - w_{Ro}) \quad \text{.....................(1a)}$$

A moisture effectivity analogous to η_{rh} in (2) is defined as

$$\eta_{rw} = \frac{w_{Lo} - w_{Li}}{w_{Ri} - w_{Li}} = \frac{w_{Ro} - w_{Ri}}{w_{Li} - w_{Ri}} \quad \text{.......................(2a)}$$

where η_{rw} is the moisture effectivity of the regenerator. Making similar assumptions to those made in the case of enthalpy effectivity, (6), the equations governing the effect of by-pass flows on overall moisture effectivity are identical to the equations governing the effect of by-pass flows on overall enthalpy effectivity, with enthalpy terms replaced by appropriate water content terms. It therefore follows that their solution is of the same form as (12) and (13), but with w replacing h.

The approach used to derive overall moisture effectivity can be applied to derive the effectivity for the transfer of any non-reactive component in the fluid streams. The resulting equations will be similar to those for moisture given above, and their solutions similar to (12) and (13).

3.3 Overall temperature effectivity:

For the temperature range 10°C to 49°C, which is that usually encountered in air conditioning regenerator systems, the data for the enthalpy of an air/water vapour mixture (Ref. 6) can be fitted by the following equation:

$$h = t + w\,(2502 + 1.8\,t) \quad \text{.......................(14)}$$

For the temperature range considered, the variation of the 1.8 t term in (14) causes only a small variation of h. Hence the enthalpy of moist air is approximated by the expression

$$h = c_a\,t + w\,h_g$$

where h_g is the enthalpy of water vapour at a fixed temperature within the range considered. We can therefore express t by the approximate relationship

$$t = \frac{1}{c_a}(h - w\,h_g)$$

Using this relationship and (1) and (1a) we obtain

$$m_r\,(t_{Lo} - t_{Li}) = m_r\,(t_{Ri} - t_{Ro}) \quad \text{.................(1b)}$$

Similarly, from (5) and (5a),

$$\begin{aligned} m_r t_{Ri} &= m_1 t_1 - m_a t_{ai} - m_c t_{ci} + m_e t_{eo} \\ m_r t_{Ro} &= m_2 t_2 - m_b t_{bo} - m_c t_{co} + m_f t_{fi} \\ m_r t_{Li} &= m_3 t_3 - m_b t_{bi} - m_d t_{di} + m_f t_{fo} \\ m_r t_{Lo} &= m_4 t_4 - m_a t_{ao} - m_d t_{do} + m_e t_{ei} \end{aligned} \quad \text{............(5b)}$$

A temperature effectivity analogous to η_{rh} and η_{rw} in (2) and (2a) is defined by

$$\eta_{rt} = \frac{t_{Lo} - t_{Li}}{t_{Ri} - t_{Li}} = \frac{t_{Ri} - t_{Ro}}{t_{Ri} - t_{Li}} \quad \text{......................(2b)}$$

Applying the assumptions expressed by (6), we find that the equations governing the effect of by-pass flows on overall temperature effectivity are identical to those for enthalpy and moisture effectivity, with temperature replacing the appropriate terms. The solution of these equations has therefore the same form as (12) and (13) but with t replacing h.

4.—PARTICULAR CASES OF BY-PASS FLOWS

A number of cases are analysed in order to understand the effects of various types and combinations of by-pass flows. Using (12) and (13), non-dimensional curves have been drawn to illustrate these effects. By-pass flow ratios from 0 to 1 are considered, though in actual systems these ratios rarely exceed 0.1.

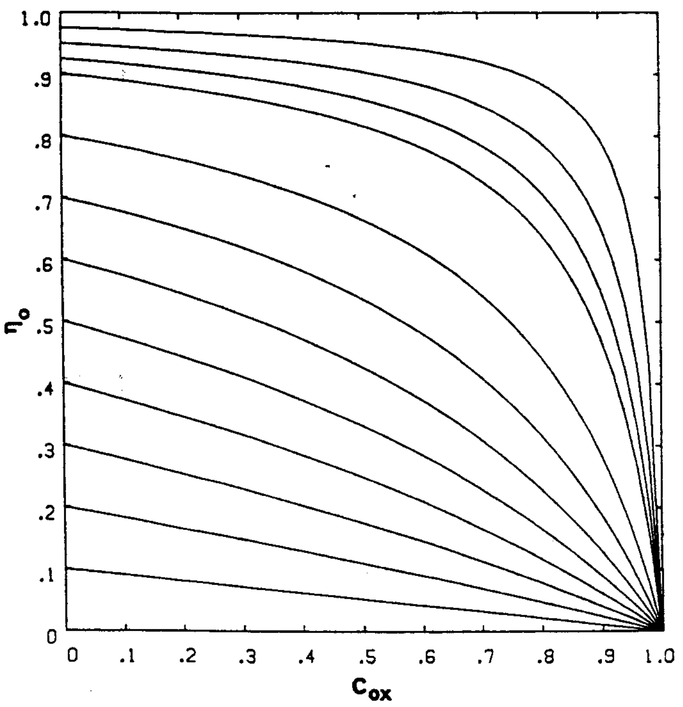

Fig. 5.—Effect of cross by-pass flow from outlet, C_{ox}.

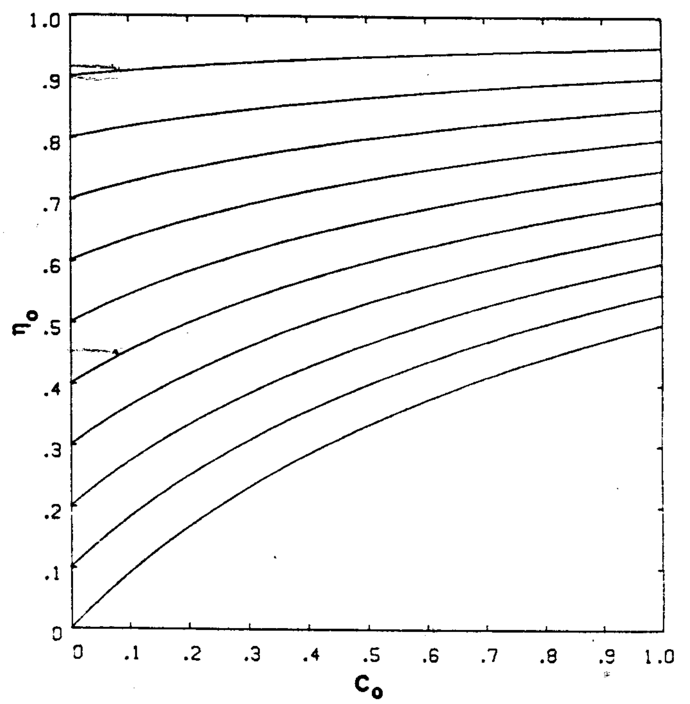

Fig. 6.—Effect of cross by-pass flow into outlet, C_o.

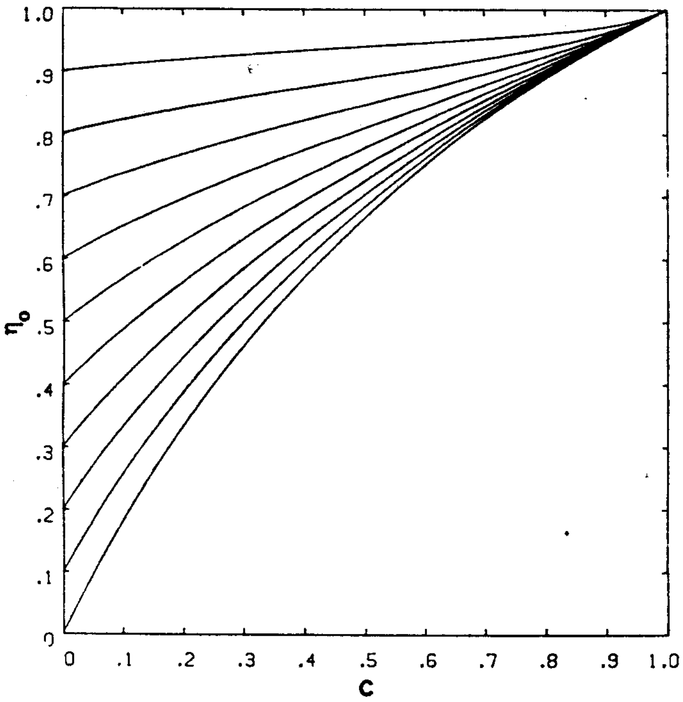

Fig. 7.—Effect of equal cross by-pass flows into inlet and outlet, $C_i = C_o = C$.

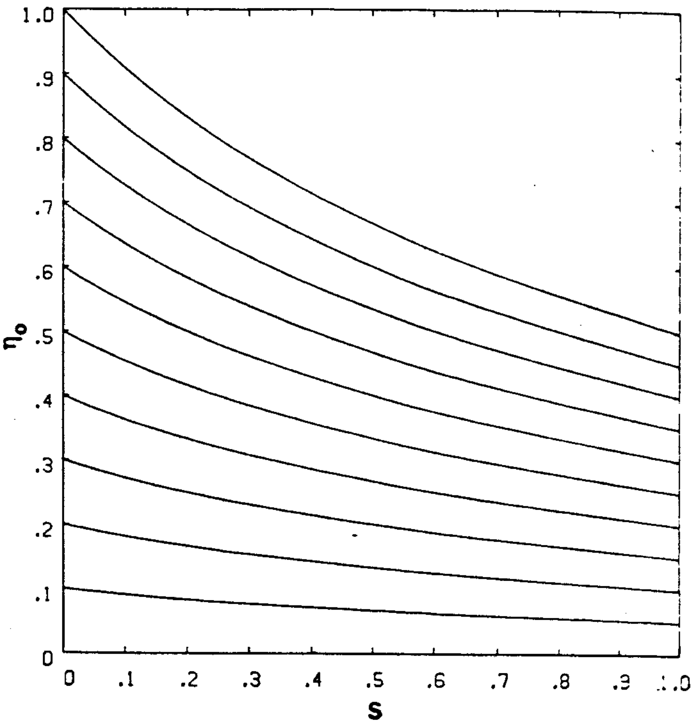

Fig. 9.—Effect of side by-pass flow into outlet, S.

Cross by-pass flow from the inlet of a side (e.g., m_b in Fig. 3) has no effect on the overall effectivity of that side, since the regenerator is assumed to remain balanced. The effect of cross by-pass flow into the inlet of a side, C_i, is to increase the effectivity of that side, Fig. 4. This by-pass flow may occur in a switched-bed regenerator due to external mixing between the main stream flows.

The effect of cross by-pass flow from the outlet of a side, C_{ox}, is to decrease the effectivity of that side, Fig. 5. This by-pass flow occurs in a rotary regenerator when the pressure level is greater on the side considered than on the other side, that is $p_4 > p_1$ when considering the left side. This case applies to a gas turbine regenerator and has been considered in Ref. 4, where the result is presented in a different manner. Fig. 6 shows the effect of cross by-pass flow into the outlet of a side, C_o, to be to increase the effectivity of that side. This by-pass flow occurs in a rotary regenerator in the common air conditioning case where $p_3 = p_1$, for which there are symmetric cross by-pass flow paths.

Fig. 7 shows the case of a rotary regenerator with equal C_i and C_o, which occurs when the pressure level is much lower on the regenerator side considered than on the other side, that is $p_2 \gg p_3$ when considering the left side. The effects shown in Figs. 4 and 6 are combined in Fig. 7.

Storage mixing in a switched-bed regenerator with equal storage volumes would give rise to C_o, C_i and C_{ox} of equal magnitude occurring simultaneously and this case is shown in Fig. 8. The effect of this combination of by-pass flows is to increase effectivity.

Side by-pass flow S, which occurs only in a rotary regenerator, decreases regenerator effectivity as shown in Fig. 9. A rotary regenerator with symmetric cross by-pass flows equal to side by-pass flows has $C_o = S$ and this case is shown in Fig. 10, which combines the effects shown in Figs. 6 and 9.

Both C_o and C_i cause an increase in effectivity, and C_{ox} and S a decrease in effectivity. When C_{ox} or C_i equals unity, the by-pass flow links the through flows to form a return loop, resulting in an impractical system.

For by-pass flow ratios appreciably less than unity, which are usual in actual systems, all figures except Fig. 9 (showing the effect of side by-pass flow S) indicate that the percentage change in effectivity caused by by-pass flow is very much larger at low than high regenerator effectivity.

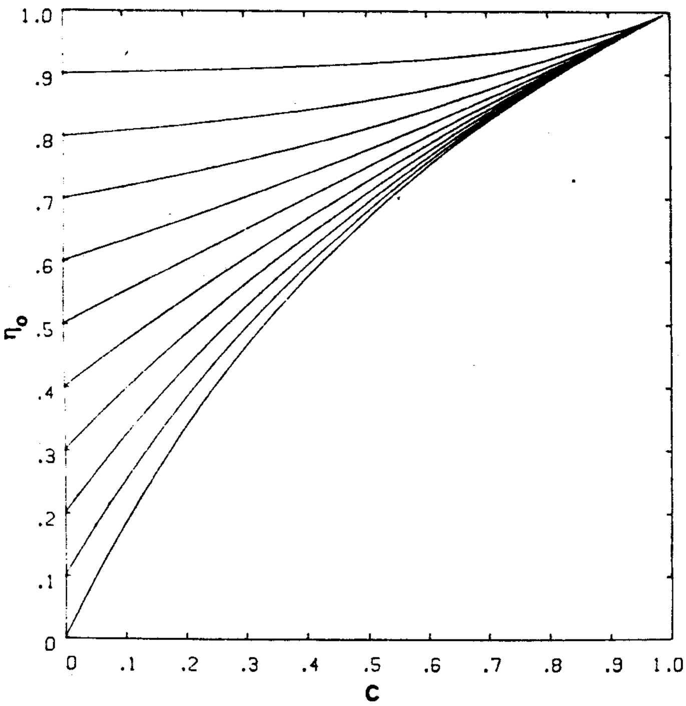

Fig. 8.—Effect of storage mixing in a switched-bed regenerator causing cross by-pass flows, $C_o = C_i = C_{ox} = C$.

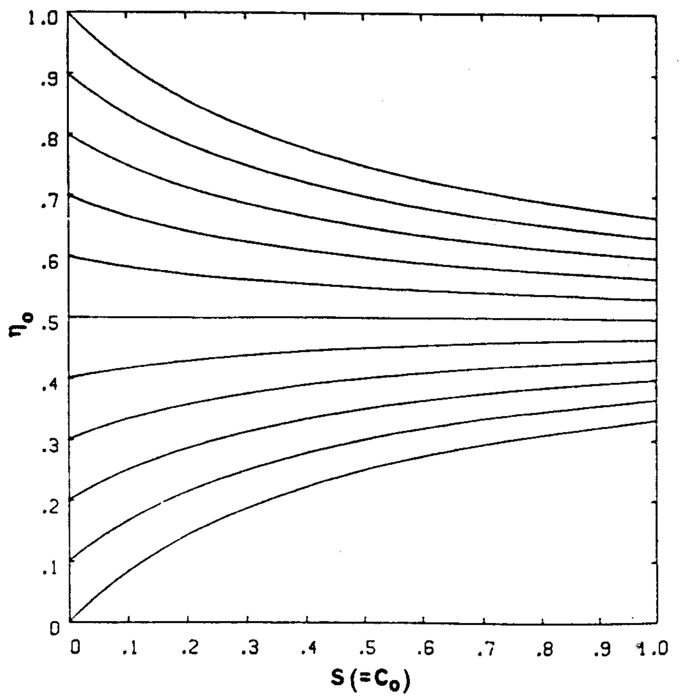

Fig. 10.—Effect of equal side and cross by-pass flows into outlet, $S = C_o$.

The matrix of an air conditioning regenerator should be designed so that the regenerator has a low effectivity for the transfer of any contaminant in the return air stream. It follows from the previous paragraph that in a regenerator so designed both C_i and C_o, Figs. 4 and 6, will have the effect of significantly increasing contaminant transfer to the fresh air stream. This conclusion applies to moisture transfer in a regenerator used for sensible heat exchange in an air conditioning system, since it is designed to have high temperature and low moisture effectivities.

5.—CONCLUSIONS

The effect of by-pass flows on regenerator performance can be readily assessed from the solutions presented.

Cross by-pass flows into the inlet and into the outlet of a regenerator side cause an increase in the overall regenerator effectivity of that side, while cross by-pass flow from the outlet of a side and side by-pass flow cause a decrease in effectivity.

For by-pass flow ratios appreciably less than unity, which are usual in actual systems, the percentage change in effectivity caused by all by-pass flows except side by-pass flow is very much larger at low than high regenerator effectivity. As a result, cross by-pass flow from the return air side of an air conditioning regenerator can be the cause of significant contaminant transfer to the fresh air stream.

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